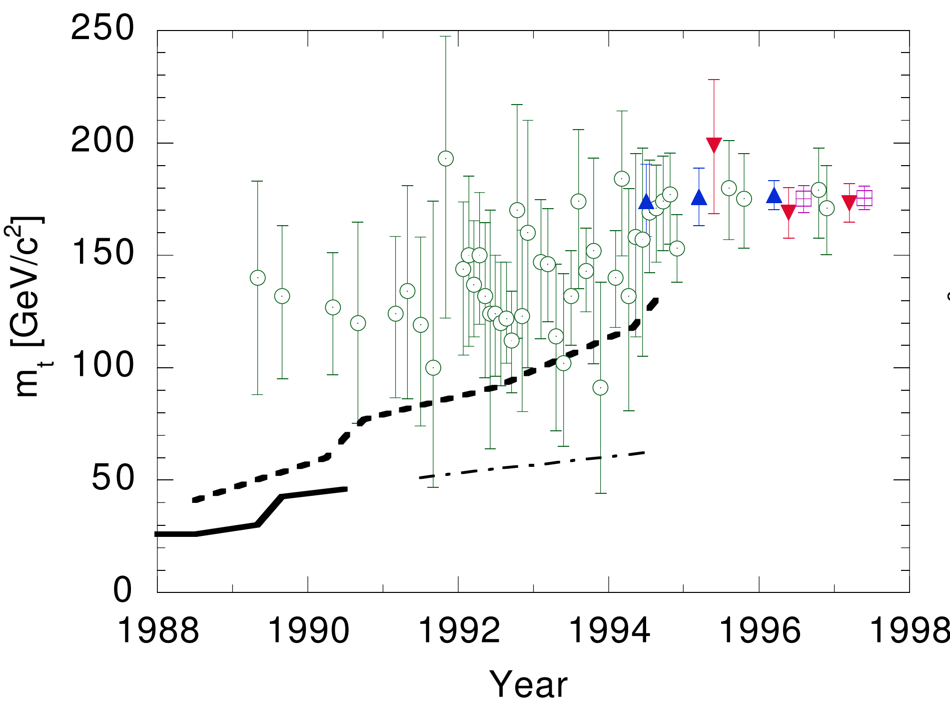
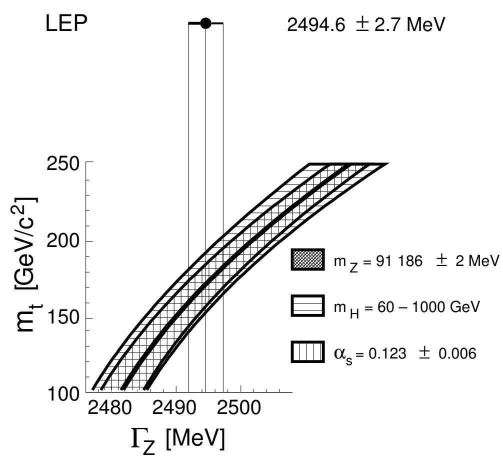
Global fits . . .

to precision EW measurements:

▷ precision improves with time

▷ calculations improve with time

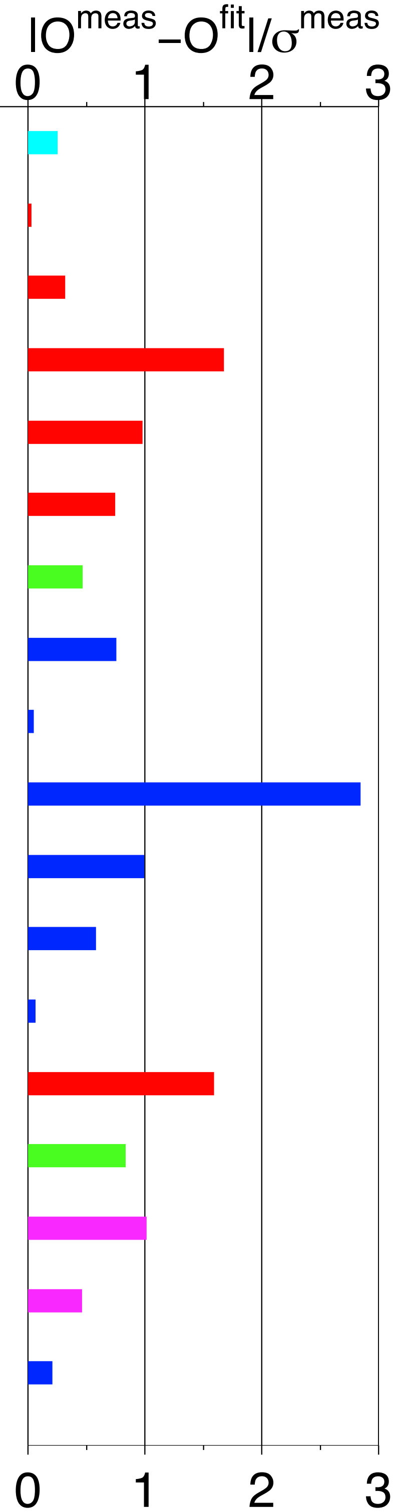




11.94, LEPEWWG: $m_t = 178 \pm 11^{+18}_{-19}$ GeV/c^2

Direct measurements: $m_t = 174.3 \pm 5.1$ GeV/c^2

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^0 [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01643
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.410 ± 0.032	80.377
Γ_W [GeV]	2.123 ± 0.067	2.092
m_t [GeV]	172.7 ± 2.9	173.3



LEP Electroweak Working Group, Summer 2005

Parity violation in atoms

Nucleon appears elementary at very low Q^2; effective Lagrangian for nucleon β-decay

$$\mathcal{L}_\beta = -\frac{G_F}{\sqrt{2}}\bar{e}\gamma_\lambda(1-\gamma_5)\nu\,\bar{p}\gamma^\lambda(1-g_A\gamma_5)n$$

$g_A \approx 1.26$: axial charge

NC interactions ($x_W \equiv \sin^2\theta_W$):

$$\begin{aligned}\mathcal{L}ep &= \frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\lambda(1-4x_W-\gamma_5)e\,\bar{p}\gamma^\lambda(1-4x_W-\gamma_5)p\,, \\ \mathcal{L}en &= \frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\lambda(1-4x_W-\gamma_5)e\,\bar{n}\gamma^\lambda(1-\gamma_5)n\end{aligned}$$

▷ Regard nucleus as a noninteracting collection of Z protons and N neutrons ▷ Perform NR reduction; nucleons contribute coherently to A_eV_N coupling, so dominant P-violating contribution to eN amplitude is

$$\mathcal{M}_{\text{pv}} = \frac{-iG_F}{2\sqrt{2}}Q^W\bar{e}\rho_N(\mathbf{r})\gamma_5 e$$

$\rho_N(\mathbf{r})$: nucleon density at e^- coordinate $\mathbf{r}$

$Q^W \equiv Z(1-4x_W) - N$: weak charge

Bennett & Wieman (Boulder): 6S-7S transition polarizability

$$\begin{aligned}Q_W(\text{Cs}) &= -72.06 \pm 0.28\ (\text{expt}) \pm 0.34\ (\text{theory}) \\ &\to -72.71 \pm 0.29\ (\text{expt}) \pm 0.39\ (\text{theory}) \\ \text{Theory} &= -73.19 \pm 0.13\end{aligned}$$

Guéna, Lintz, Bouchiat, *Mod. Phys. Lett. A* **20,** 375 (2005)

The vacuum energy problem

Higgs potential $V(\varphi^\dagger\varphi) = \mu^2(\varphi^\dagger\varphi) + |\lambda|\,(\varphi^\dagger\varphi)^2$

At the minimum,

$$V(\langle\varphi^\dagger\varphi\rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda|\,v^4}{4} < 0.$$

Identify $M_H^2 = -2\mu^2$

contributes field-independent vacuum energy density

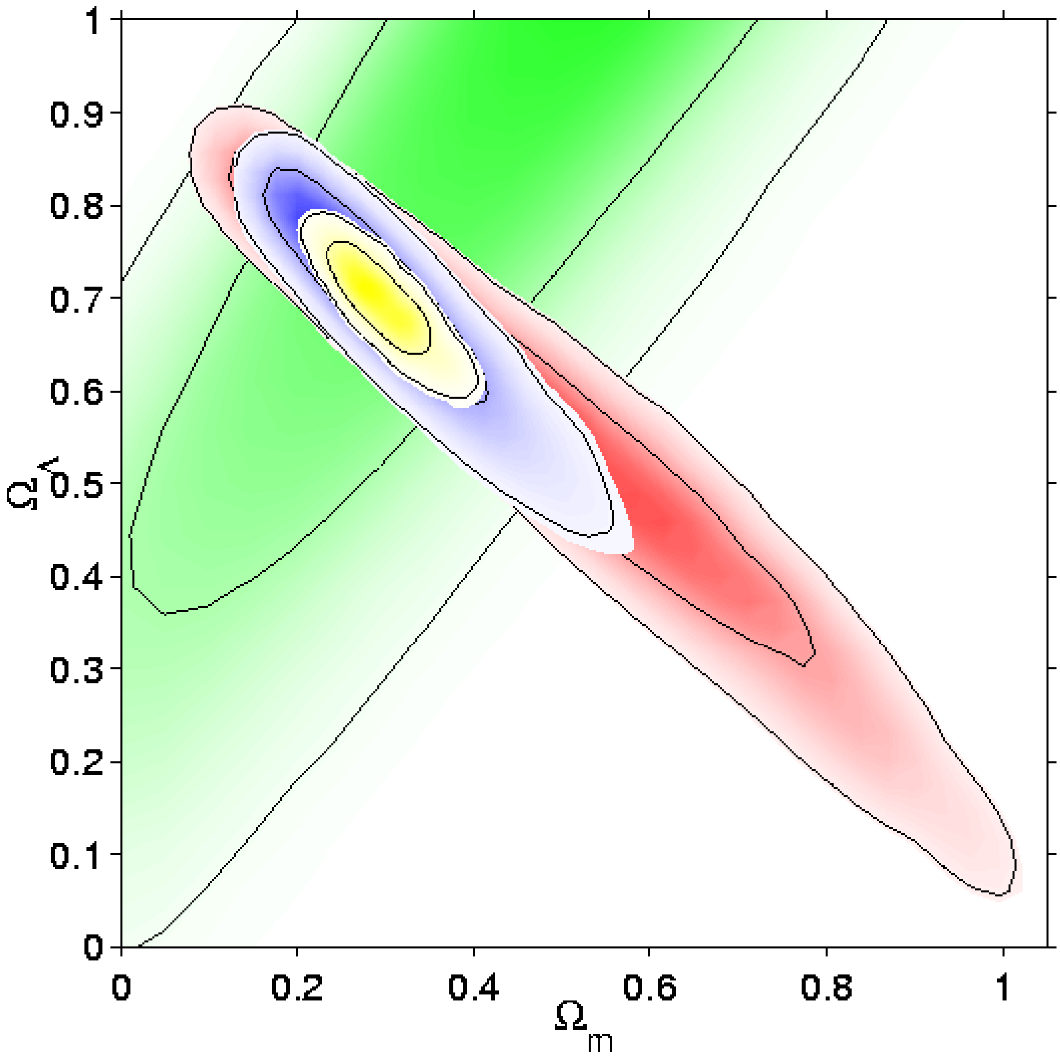
$$\boxed{\varrho_H \equiv \frac{M_H^2 v^2}{8}}$$

Adding vacuum energy density $\varrho_{\text{vac}} \Leftrightarrow$ adding cosmological constant Λ to Einstein's equation

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = \frac{8\pi G_N}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$$\Lambda = \frac{8\pi G_N}{c^4}\varrho_{\text{vac}}$$

observed vacuum energy density $\varrho_{\text{vac}} \lesssim 10^{-46}$ GeV^4



Lewis & Bridle, astro-ph/0205436

But $M_H \gtrsim 114$ GeV/c^2 $\Rightarrow$

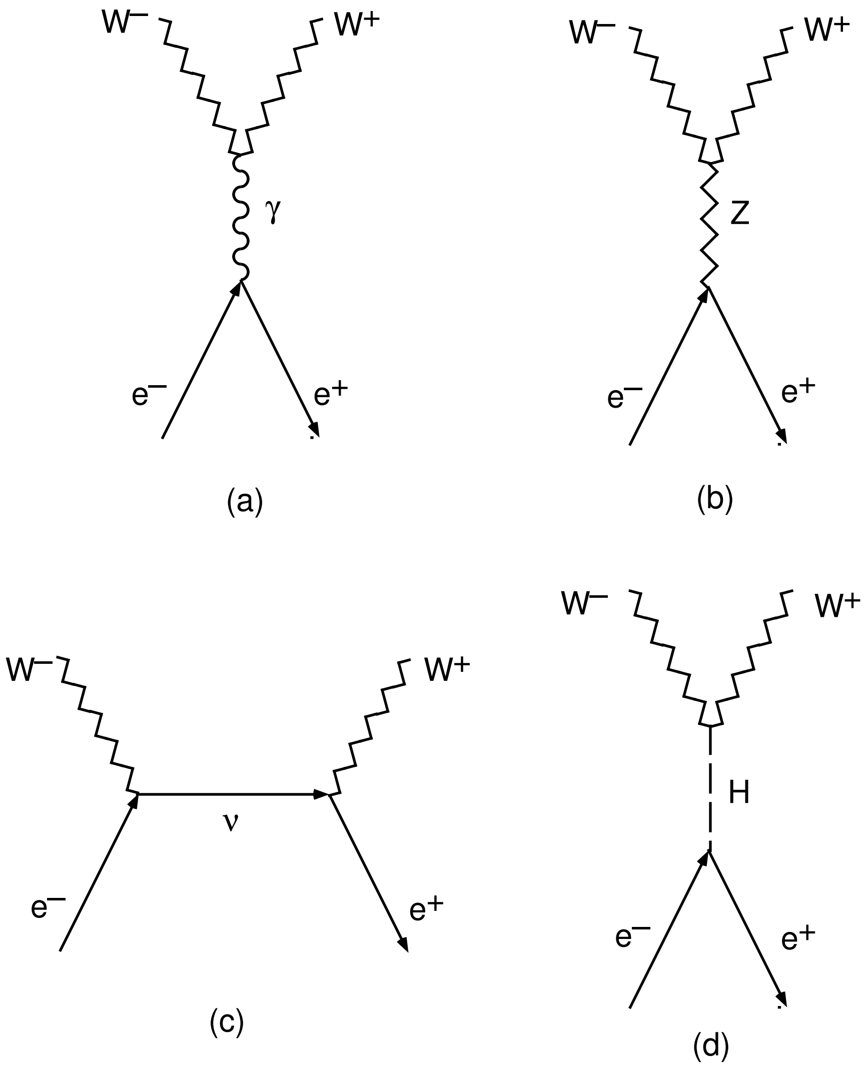
$$\varrho_H \gtrsim 10^8 \text{ GeV}^4$$

MISMATCH BY 54 ORDERS OR MAGNITUDE

Why a Higgs Boson Must Exist

▷ Role in canceling high-energy divergences

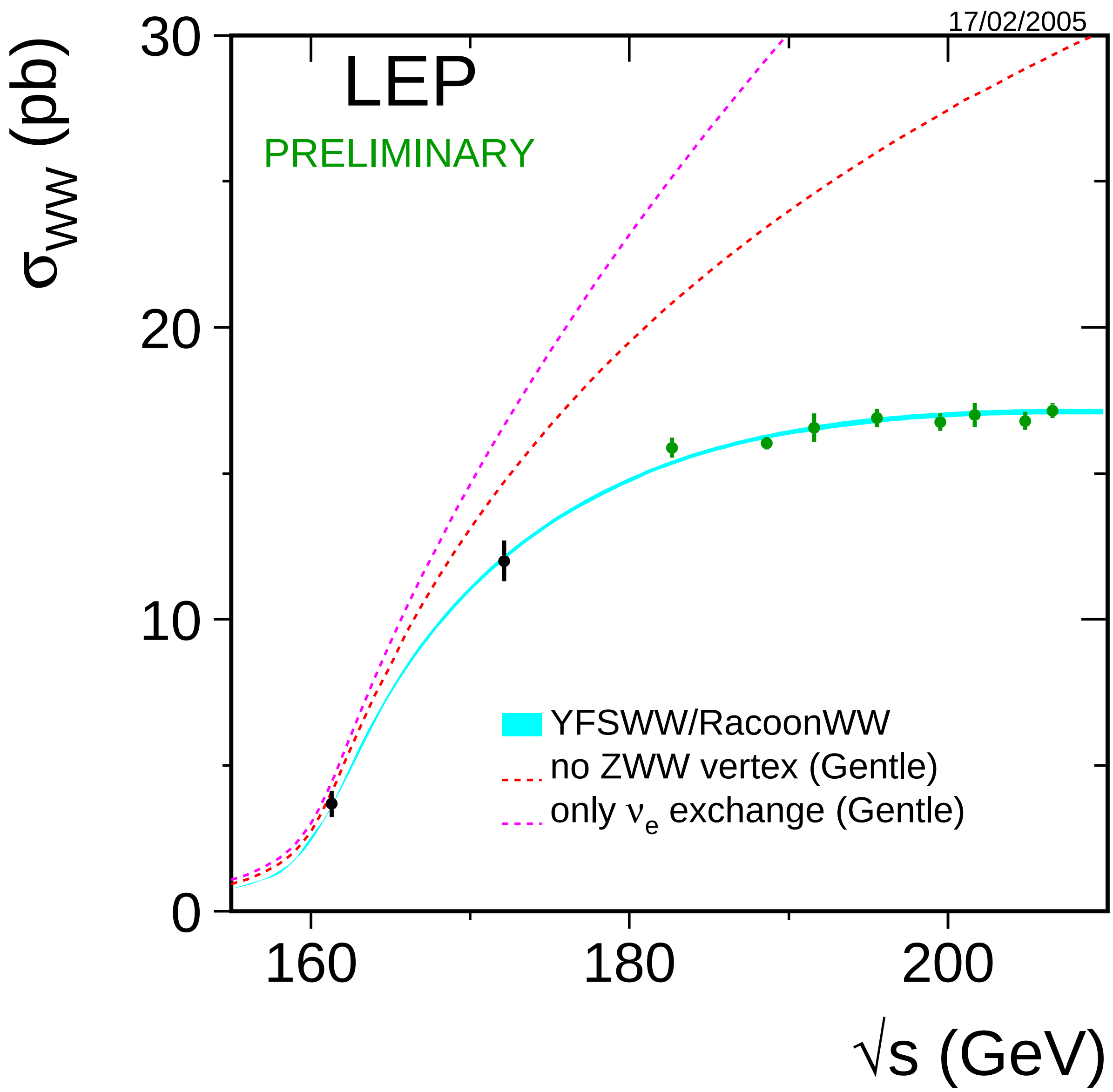
S-matrix analysis of $e^+e^- \to W^+W^-$



$J = 1$ partial-wave amplitudes $\mathcal{M}_\gamma^{(1)}$, $\mathcal{M}_Z^{(1)}$, $\mathcal{M}_\nu^{(1)}$ have—individually—unacceptable high-energy behavior $(\propto s)$

. . . But sum is well-behaved

"Gauge cancellation" observed at LEP2, Tevatron



$J = 0$ amplitude exists because electrons have mass, and can be found in "wrong" helicity state

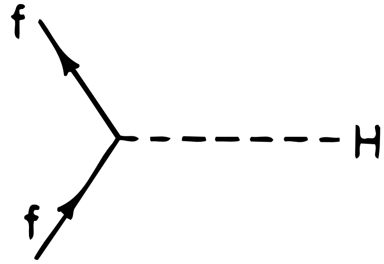
$$\mathcal{M}_{\nu}^{(0)} \propto s^{\frac{1}{2}}$$: unacceptable HE behavior

(no contributions from γ and Z)

This divergence is canceled by the Higgs-boson contribution

$\Rightarrow He\bar{e}$ coupling must be $\propto m_e$,

because "wrong-helicity" amplitudes $\propto m_e$



$$\frac{-im_f}{v} = -im_f(G_F\sqrt{2})^{1/2}$$

If the Higgs boson did not exist, *something else* would have to cure divergent behavior

IF gauge symmetry were unbroken . . .

▷ no Higgs boson

▷ no longitudinal gauge bosons

▷ no extreme divergences

▷ no wrong-helicity amplitudes

. . . and no viable low-energy phenomenology

In spontaneously broken theory . . .

▷ gauge structure of couplings eliminates the most severe divergences

▷ lesser—but potentially fatal—divergence arises because the electron has mass

. . . due to the Higgs mechanism

▷ SSB provides its own cure—the Higgs boson

A similar interplay and compensation *must exist* in any acceptable theory

Bounds on M_H

EW theory does not predict Higgs-boson mass

Self-consistency ⇒ plausible lower and upper bounds

▷ Conditional *upper bound* from Unitarity

Compute amplitudes $\mathcal{M}$ for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta)$$

Most channels decouple—pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies)—for any M_H.

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0/\sqrt{2} \quad HH/\sqrt{2} \quad HZ_L^0$$

L: longitudinal, $1/\sqrt{2}$ for identical particles

In HE limit,[a] s-wave amplitudes $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2} (a_0) \to \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect the partial-wave unitarity condition $|a_0| \leq 1$

$$\Longrightarrow \quad M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F}\right)^{1/2} = 1 \text{ TeV}/c^2$$

condition for perturbative unitarity

[a]Convenient to calculate using *Goldstone-boson equivalence theorem*, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by $\mathcal{L}_{\text{int}} = -\lambda v h(2w^+w^- + z^2 + h^2) - (\lambda/4)(2w^+w^- + z^2 + h^2)^2$, with $1/v^2 = G_F\sqrt{2}$ and $\lambda = G_F M_H^2/\sqrt{2}$.

▷ If the bound is respected

⋆ weak interactions remain weak at all energies

⋆ perturbation theory is everywhere reliable

▷ If the bound is violated

⋆ perturbation theory breaks down

⋆ weak interactions among $W^{\pm}$, Z, and H become strong on the 1-TeV scale

$\Rightarrow$ features of *strong* interactions at GeV energies will characterize *electroweak* gauge boson interactions at TeV energies

Threshold behavior of the pw amplitudes a_{IJ} follows from chiral symmetry

$$\begin{array}{lll} a_{00} \approx & G_F s/8\pi\sqrt{2} & \text{attractive} \\ a_{11} \approx & G_F s/48\pi\sqrt{2} & \text{attractive} \\ a_{20} \approx & -G_F s/16\pi\sqrt{2} & \text{repulsive} \end{array}$$

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV

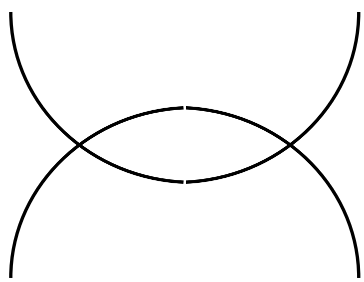
▷ Triviality of scalar field theory

Only *noninteracting* scalar field theories make sense on all energy scales

Quantum field theory vacuum is a dielectric medium that screens charge ⇒ *effective charge* is a function of the distance or, equivalently, of the energy scale

running coupling constant

In $\lambda\phi^4$ theory, it is easy to calculate the variation of the coupling constant λ in perturbation theory by summing bubble graphs



$\lambda(\mu)$ is related to a higher scale Λ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2}\log\left(\Lambda/\mu\right)$$

(Perturbation theory reliable only when λ is small, lattice field theory treats strong-coupling regime)

For stable Higgs potential (*i.e.*, for vacuum energy not to race off to $-\infty$), *require* $\lambda(\Lambda) \geq 0$

Rewrite RGE as an inequality

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log\left(\Lambda/\mu\right) \; .$$

implies an *upper bound*

$$\boxed{\lambda(\mu) \leq 2\pi^2/3\log\left(\Lambda/\mu\right)}$$

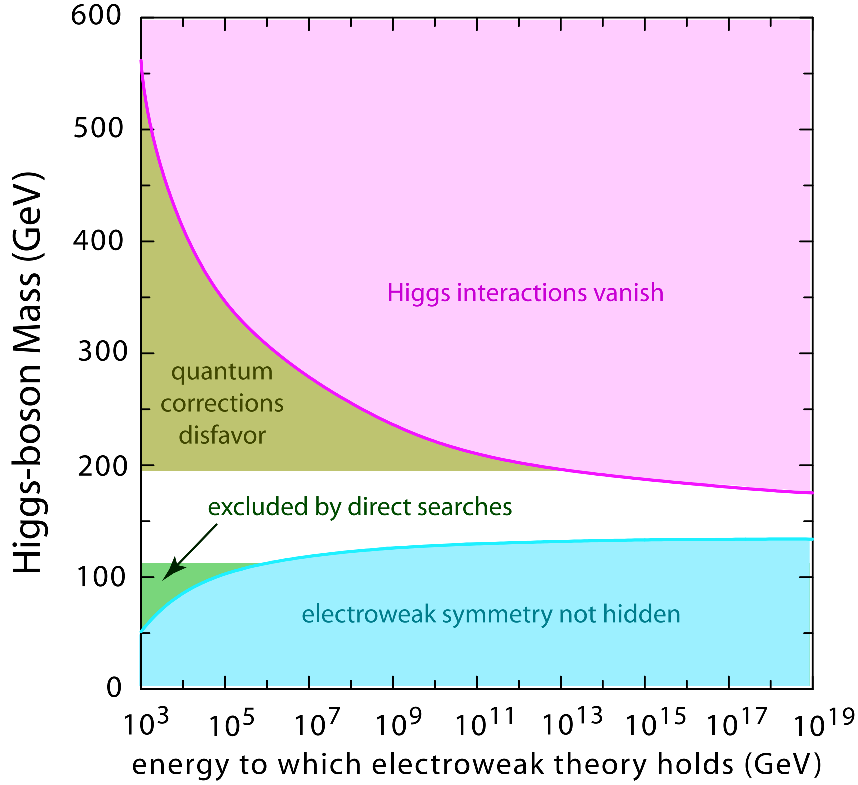
If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit $\Lambda \to \infty$ while holding μ fixed at some reasonable physical scale. In this limit, the bound forces $\lambda(\mu)$ to zero. $\longrightarrow$ free field theory "trivial"

Rewrite as bound on M_H:

$$\Lambda \leq \mu \exp\left(\frac{2\pi^2}{3\lambda(\mu)}\right)$$

Choose $\mu = M_H$, and recall $M_H^2 = 2\lambda(M_H)v^2$

$$\boxed{\Lambda \leq M_H \exp\left(4\pi^2 v^2/3M_H^2\right)}$$



Moral: For any M_H, there is a *maximum energy scale* $\Lambda^\star$ at which the theory ceases to make sense.

The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies

Perturbative analysis breaks down when $M_H \to 1$ TeV/c^2 and interactions become strong

Lattice analyses $\Longrightarrow M_H \lesssim 710 \pm 60$ GeV/c^2 if theory describes physics to a few percent up to a few TeV

If $M_H \to 1$ TeV EW theory lives on brink of instability

▷ *Lower bound* by requiring EWSB vacuum $V(v) < V(0)$

Requiring that $\langle\phi\rangle_0 \neq 0$ be an absolute minimum of the one-loop potential up to a scale Λ yields the vacuum-stability condition

$$M_H^2 > \frac{3G_F\sqrt{2}}{8\pi^2}(2M_W^4 + M_Z^4 - 4m_t^4)\log(\Lambda^2/v^2)$$

... for $m_t \lesssim M_W$

(No illuminating analytic form for heavy m_t)

If the Higgs boson is relatively light—which would itself require explanation—then the theory can be self-consistent up to very high energies

If EW theory is to make sense all the way up to a unification scale $\Lambda^\star = 10^{16}$ GeV, then

$$134 \text{ GeV}/c^2 \lesssim M_H \lesssim 177 \text{ GeV}/c^2$$

Higgs-Boson Properties

$$\Gamma(H \to f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

$\propto M_H$ in the limit of large Higgs mass

$$\Gamma(H \to W^+W^-) = \frac{G_F M_H^3}{32\pi\sqrt{2}}(1-x)^{1/2}(4-4x+3x^2)$$

$$x \equiv 4M_W^2/M_H^2$$

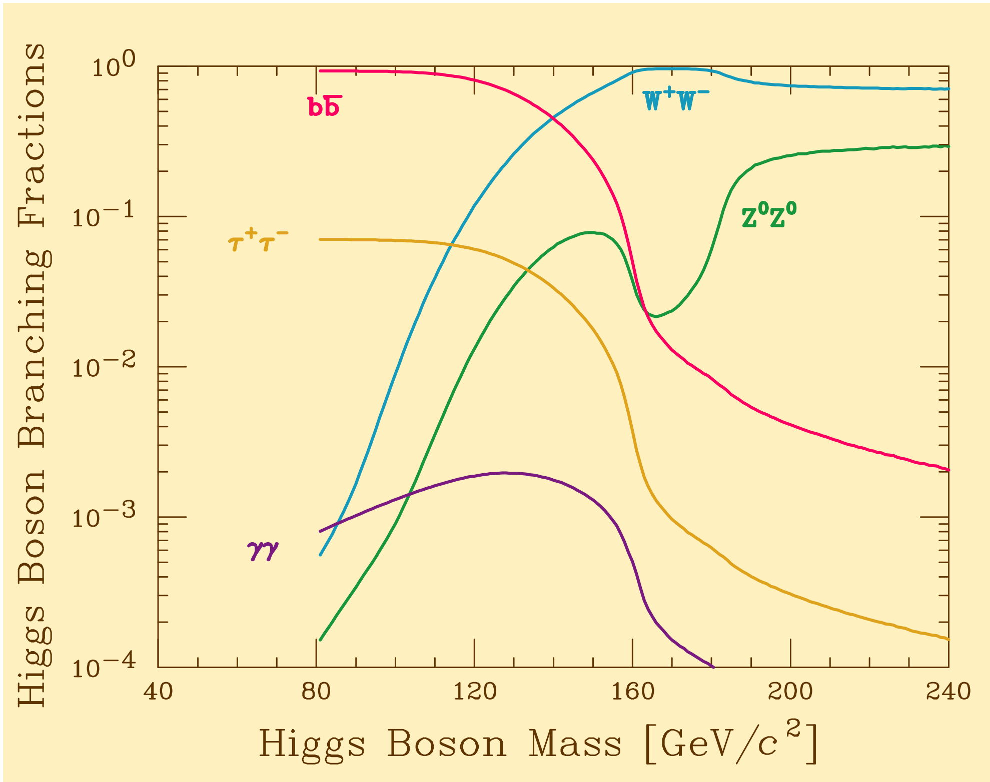
$$\Gamma(H \to Z^0Z^0) = \frac{G_F M_H^3}{64\pi\sqrt{2}}(1-x')^{1/2}(4-4x'+3x'^2)$$

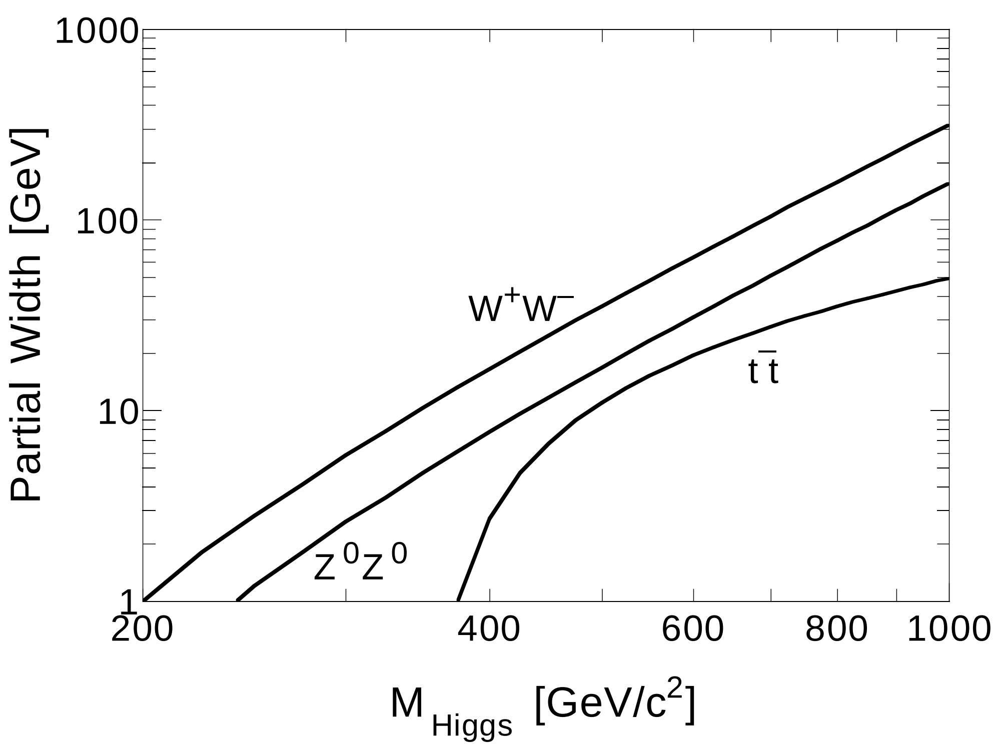
$$x' \equiv 4M_Z^2/M_H^2$$

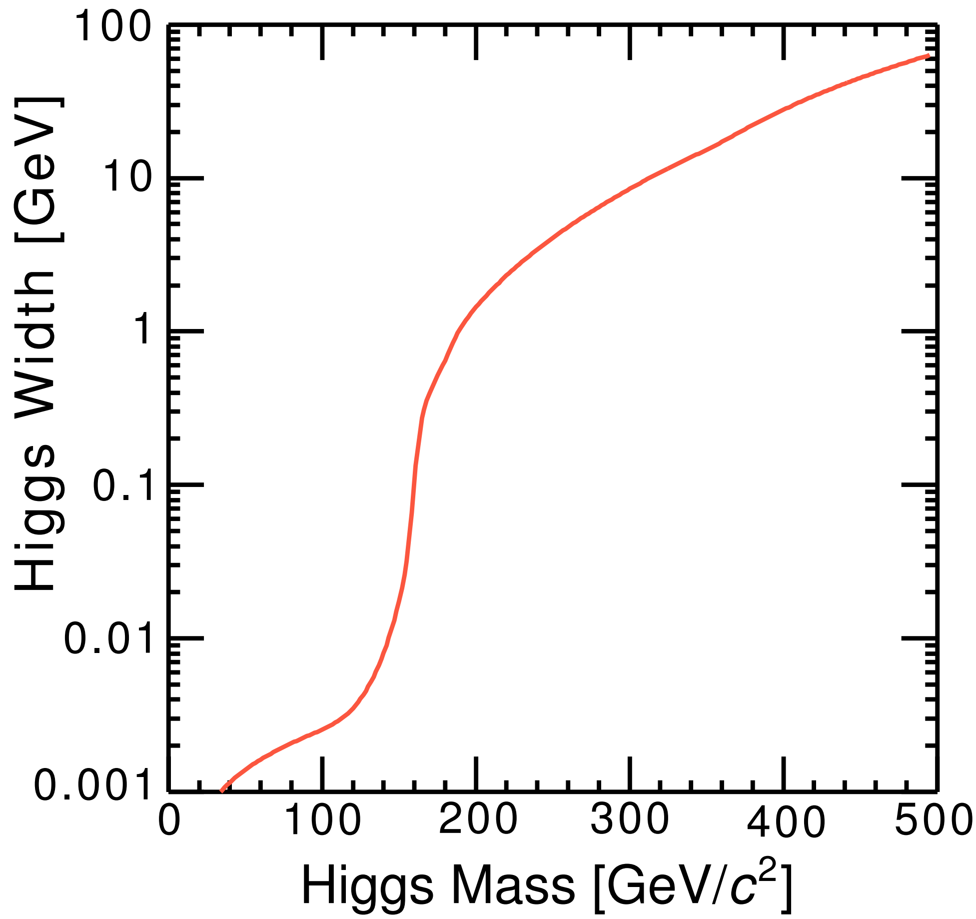
asymptotically $\propto M_H^3$ and $\frac{1}{2}M_H^3$, respectively
($\frac{1}{2}$ from weak isospin)

$2x^2$ and $2x'^2$ terms $\Leftrightarrow$ decays into transversely polarized gauge bosons

Dominant decays for large M_H into pairs of longitudinally polarized weak bosons


Higgs Boson Branching Fractions
Higgs Boson Mass [GeV/c²]
bb̄
W⁺W⁻
Z⁰Z⁰
τ⁺τ⁻
γγ
10⁰
10⁻¹
10⁻²
10⁻³
10⁻⁴
40
80
120
160
200
240


Partial Width [GeV]
M Higgs [GeV/c²]
W⁺W⁻
Z⁰Z⁰
t t̄
1000
100
10
1
200
400
600
800
1000



Below W^+W^- threshold, $\Gamma_H \lesssim 1$ GeV

Far above W^+W^- threshold, $\Gamma_H \propto M_H^3$

For $M_H \to 1$ TeV/c^2, Higgs boson is an *ephemeron*, with a perturbative width approaching its mass.